Galois Theory Mid Term

September

This exam is of **30 marks** and is **3 hours long**. Please **read all the questions carefully**. Please feel free to use whatever theorems you have learned in class after stating them clearly.

1. Let N/F be a Galois extension with $Gal(N/F) \simeq \mathfrak{A}_4$, the alternating subgroup of the symmetric group on 4 elements, $\mathfrak{S}_4$. Show that there does not exists a subfield L with $[N : L] = 2$. (4)

2. Let F be a field. Does there exists an extension K/F such that $[K^{Gal(K/F)} : F] = 2$? If so, give an example. (4).

3. Answer **True** or **False**. If **True**, prove it, if **False**, give a counterexample. Let F, L and K be fields.

   1. If L/F and K/F are **normal**, then KL/F is **normal**. (3)
   2. If KL/F is **normal** then K/F and L/F are **normal**. (3)
   3. If L/F and K/F are **separable**, then KL/F is **separable**. (3)
   4. If KL/F is **separable** then K/F and L/F are **separable** (3)
   5. If L/F and K/F are **purely inseparable**, then KL/F is **purely inseparable**. (3)
   6. If KL/F is **purely inseparable** and L/F and K/F are **purely inseparable**. (3)

4. Let K be the rational function field $k(x)$ over a perfect field k of $char(k) = p$. Let $F = k(u)$ for some $u \in K$, $u = f/g$. Show that K/F is separable if and only if $u \notin K^p$. (4)